

# Classifying the behavior of noncanonical quintessence

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We derive general conditions for the existence of stable scaling solutions for the evolution of noncanonical quintessence, with a Lagrangian of the form  $\mathcal{L}(X, \phi) = X^\alpha - V(\phi)$ , for power-law and exponential potentials when the expansion is dominated by a background barotropic fluid. Our results suggest that in most cases, noncanonical quintessence with such potentials does not yield interesting models for the observed dark energy. When the scaling solution is not an attractor, there is a wide range of model parameters for which the evolution asymptotically resembles a zero-potential solution with equation of state parameter  $w = 1/(2\alpha - 1)$ , and oscillatory solutions are also possible for positive power-law potentials; we derive the conditions on the model parameters which produce both types of behavior. We investigate thawing noncanonical models with a nearly-flat potential and derive approximate expressions for the evolution of  $w(a)$ . These forms for  $w(a)$  differ in a characteristic way from the corresponding expressions for canonical quintessence.

PACS numbers: 98.80.Cq

## I. INTRODUCTION

Observational evidence [1–7] indicates that approximately 70% of the energy density in the universe is in the form of a negative-pressure component, called dark energy, with the remaining 30% in the form of nonrelativistic matter. The dark energy component can be characterized by its equation of state parameter,  $w$ , defined as the ratio of the dark energy pressure to its density:

$$w = p/\rho, \quad (1)$$

where a cosmological constant,  $\Lambda$ , corresponds to  $w = -1$  and  $\rho = \text{constant}$ .

While a model with a cosmological constant and cold dark matter ( $\Lambda$ CDM) is consistent with current observations, there are many models for dark energy that predict a dynamical equation of state. The most widely studied of these are quintessence models, in which the dark energy arises from a time-dependent scalar field,  $\phi$  [8–14]. (See Ref. [15] for a review).

In this paper, we examine quintessence models with the noncanonical Lagrangian of the form

$$\mathcal{L}(X, \phi) = X^\alpha - V(\phi), \quad (2)$$

where  $X \equiv \dot{\phi}^2/2$ , and we take  $\hbar = c = 8\pi G = 1$  throughout. This model has been examined both as a model for inflation [16, 17] and for dark energy [18–22]. Refs. [21, 22] are the previous papers most closely related to our discussion. In Ref. [21], Sahni and Sen examined a model described by Eq. (2) for which  $V(\phi)$  is a constant, and they argued that for  $\alpha \gg 1$ , the scalar field can account for both the dark matter and dark energy. In Ref. [22], Ossoulian et al. examined power-law models for  $V(\phi)$  and provided cases for which the resulting evolution

is stable. We extend and generalize this previous work by deriving the exact conditions for stability for power law potentials, extending this discussion to exponential potentials, and examining noncanonical quintessence in the limit where the potential is nearly flat and the model is close to  $\Lambda$ CDM.

Canonical quintessence models with power-law or exponential potentials were among the first to be investigated [8, 9], but these models have serious problems in fitting current observations. For instance, for a potential of the form  $V(\phi) \propto \phi^{-n}$ , the data require  $n < 1$  [23, 24], which is not particularly natural. Exponential potentials fare even worse. They can lead to “tracker” solutions, in which  $w$  for the scalar field tracks the the same value as  $w$  for the background fluid [9–11]. While such models are interesting, they are not consistent with observations. On the other hand, power-law and exponential potentials arise naturally in the context of many different models, so it is important to see whether the noncanonical models examined here can resurrect them as viable quintessence models.

We also investigate the behavior of noncanonical models when the potential is nearly flat and the universe contains both quintessence and nonrelativistic matter. Models of this kind, in which the scalar field is initially at rest (“thawing” models [26]), are a natural way to produce  $w$  near  $-1$ , in agreement with observations. Scherrer and Sen [25] showed that such models converge toward a similar evolution at late times independent of the details of the potential; it is interesting to determine whether this is also the case for noncanonical quintessence.

In the next section, we rederive solutions for the power-law potential with an expansion dominated by a background barotropic fluid, and we determine the general conditions for stability. We then perform the same cal-

culuation for the exponential potential. In Sec. III, we derive an approximation for the equation of state when the potential is nearly flat, and we discuss the cosmological implications of all of our results in Sec. IV.

## II. BACKGROUND-DOMINATED EVOLUTION

### A. Basic Formalism

We work throughout with the Lagrangian given by Eq. (2). The sound speed in this model is [19]

$$c_s^2 = \frac{1}{2\alpha - 1}. \quad (3)$$

To ensure that  $0 \leq c_s^2 \leq 1$ , we will always assume that  $\alpha \geq 1$ . (For a more detailed discussion of this point, see Ref. [27]). The energy density and pressure for  $\phi$  are given by:

$$\rho_\phi = (2\alpha - 1)X^\alpha + V(\phi), \quad (4)$$

$$p_\phi = X^\alpha - V(\phi). \quad (5)$$

In a spatially-flat Friedmann-Robertson-Walker Universe the equation of motion for  $\phi$  is

$$(\dot{\phi}^{2\alpha-1})^\cdot + 3H\dot{\phi}^{2\alpha-1} + \frac{2^{\alpha-1}}{\alpha} \frac{dV}{d\phi} = 0, \quad (6)$$

which can also be written as

$$\ddot{\phi} + \frac{3H\dot{\phi}}{2\alpha - 1} + \frac{V'(\phi)}{\alpha(2\alpha - 1)} \left( \frac{2}{\dot{\phi}^2} \right)^{\alpha-1} = 0. \quad (7)$$

Consider a universe dominated by a background barotropic fluid such as nonrelativistic matter or energy, characterized by an equation of state parameter  $w_B$  (where, e.g.,  $w_B = 0$  for nonrelativistic matter and  $w_B = 1/3$  for radiation). In this case, the background density scales as

$$\rho \propto a^{-m}, \quad (8)$$

where

$$m = 3(1 + w_B). \quad (9)$$

To simplify our expressions, we will work with  $m$  instead of  $w_B$ . Then  $m = 3$  for the matter-dominated epoch and  $m = 4$  when the universe is radiation-dominated. The Hubble parameter is then given by

$$H = \frac{2}{mt}. \quad (10)$$

In the derivations that follow, we always take  $0 \leq m \leq 6$ , corresponding to  $-1 \leq w_B \leq 1$ .

### B. Power Law Potential

Consider first the power-law potential

$$V(\phi) = V_0 \phi^n, \quad (11)$$

where  $n$  can be positive or negative. Then Eq. (6) becomes

$$(\dot{\phi}^{2\alpha-1})^\cdot + \frac{6}{mt} \dot{\phi}^{2\alpha-1} + \frac{2^{\alpha-1} n V_0}{\alpha} \phi^{n-1} = 0. \quad (12)$$

Assuming a solution of the form

$$\phi = Ct^\gamma, \quad (13)$$

the coefficient and power index are found to be:

$$\gamma = \frac{2\alpha}{2\alpha - n}, \quad (14)$$

$$C^{m-2\alpha} = - \left[ (2\alpha - 1) + (2\alpha - n) \frac{6}{mn} \right] \left( \frac{\gamma^{2\alpha}}{V_0 2^\alpha} \right)$$

These results are a rederivation of those in Ref. [22]. Note, however, that in order for the solution to be well-defined, the right-hand side of Eq. (14) must be positive. The second factor is manifestly positive, which means that

$$(2\alpha - 1) + (2\alpha - n) \frac{6}{mn} < 0. \quad (15)$$

The conditions for this inequality to be satisfied depend on the value of  $(2\alpha - 1)m$ . For  $(2\alpha - 1)m < 6$  (which includes the canonical case), we have

$$n < 0 \quad \text{or} \quad n > \frac{12\alpha}{6 - (2\alpha - 1)m}, \quad (16)$$

while for  $(2\alpha - 1)m > 6$ , the condition for the scaling solution to exist is

$$- \frac{12\alpha}{(2\alpha - 1)m - 6} < n < 0. \quad (17)$$

Substituting the above scaling solution into Eqs. (4)-(5), we obtain the equation of state parameter

$$1 + w = - \frac{\alpha mn}{3(2\alpha - n)}. \quad (18)$$

which reduces to the canonical result for  $\alpha = 1$ .

Now we must determine the parameter ranges over which our solutions represent stable attractors. We will follow the methods used previously in Refs. [9, 13]. We define the variables

$$\begin{aligned} x_1 &= \phi, \\ x_2 &= \dot{\phi}^{2\alpha-1}, \end{aligned} \quad (19)$$

and Eq. (12) is transformed into the following form:

$$\begin{aligned} \dot{x}_1 &= x_2^{1/(2\alpha-1)} \\ \dot{x}_2 &= - \frac{6}{mt} x_2 - \frac{n 2^{\alpha-1} V_0}{\alpha} x_1^{n-1}. \end{aligned} \quad (20)$$

Making the change of variables

$$u_1 = \frac{x_1}{x_e} - 1, \quad (21)$$

$$u_2 = \frac{x_2}{\dot{x}_e^{2\alpha-1}} - 1, \quad (22)$$

$$t = e^\tau, \quad (23)$$

where  $x_e = Ct^\gamma$  is the exact solution, we obtain:

$$\begin{aligned} u_1' &= -\gamma u_1 + \gamma(1 + u_2)^{1/(2\alpha-1)} - \gamma, \\ u_2' &= -Bu_2 + B(1 + u_1)^{n-1} - B, \end{aligned} \quad (24)$$

where  $B = 6/m + (\gamma - 1)(2\alpha - 1)$ , and the prime denotes the derivative with respect to  $\tau$ . We linearize Eqs. (24) about the critical point at  $u_1 = u_2 = 0$  and solve for the eigenvalues  $\Delta_\pm$  of small perturbations about this point:

$$\Delta_\pm = \frac{1}{2} \left[ -(\gamma + B) \pm \sqrt{(\gamma + B)^2 + 4\gamma B \frac{n-2\alpha}{2\alpha-1}} \right]. \quad (25)$$

Stability then requires that the real part of both eigenvalues should be negative. Ref. [22] previously derived these eigenvalues, but here we determine the exact conditions on  $\alpha$ ,  $m$ , and  $n$  for which the solutions are stable.

The stability condition is divided into two cases, depending on the values of  $m$  and  $\alpha$ . Taking  $\alpha \geq 1$  and  $0 \leq m \leq 6$ , we find, after some tedious algebra, the following stability conditions:

For  $(2\alpha - 1)m < 6$ :

$$n < 2\alpha \quad \text{or} \quad n > \frac{2\alpha(m+6)}{6 - (2\alpha - 1)m}. \quad (26)$$

For  $(2\alpha - 1)m > 6$ :

$$-\frac{12\alpha}{(2\alpha - 1)m - 6} < n < 2\alpha. \quad (27)$$

We must now combine these results with the existence conditions given in Eqs. (16) and (17) to derive the final conditions on  $\alpha$ ,  $m$ , and  $n$  that yield stable scaling solutions. Our final result is the following set of conditions:

For  $(2\alpha - 1)m < 6$ :

$$n < 0 \quad \text{or} \quad n > \frac{2\alpha(m+6)}{6 - (2\alpha - 1)m}. \quad (28)$$

For  $(2\alpha - 1)m > 6$ :

$$-\frac{12\alpha}{(2\alpha - 1)m - 6} < n < 0. \quad (29)$$

Eq. (28) reduces to the results of Ref. [13] for the case of  $\alpha = 1$ .

When the lower bound in Eq. (29) is violated, we find a new set of late-time attractor solutions. These are given by

$$\dot{\phi}^{2\alpha-1} \propto t^{-6/m} \quad (30)$$

with

$$w = \frac{1}{2\alpha - 1}. \quad (31)$$

It is easy to see that the expression for  $\phi$  corresponding to Eq. (30) is a solution of Eq. (12) in the limit where the third term in Eq. (12) is negligible (i.e., the potential is nearly constant). However, when the lower bound in Eq. (29) is violated, this third term decays away more rapidly with time than the second term, so that this is, in fact, the correct asymptotic solution.

The solution given by Eqs. (30) and (31) is identical to the constant-potential solution derived by Sahni and Sen [21] for the special case where  $V(\phi) = 0$ . Our results indicate that their model does not necessarily require an exactly flat potential; it can be achieved by a sufficiently rapidly decaying potential; i.e., one for which  $n < -12\alpha/[(2\alpha - 1)m - 6]$  (although the model of Ref. [21] in this case would also require the addition of a constant to this potential).

There is also another mode of evolution when the solution is no longer an attractor: for  $n > 0$  and  $n$  even, the potential can support oscillatory solutions. These occur more generally for arbitrary  $n$  and potentials of the form

$$V(\phi) = V_0|\phi|^n. \quad (32)$$

Oscillating canonical scalar fields were first investigated by Turner [28], and later reexamined by many others (see Ref. [29] and references therein). Unnikrishnan et al. [16] examined oscillating noncanonical scalar fields and showed that the period-averaged equation of state parameter is given by:

$$\langle w \rangle = \frac{n - 2\alpha}{n(2\alpha - 1) + 2\alpha}. \quad (33)$$

Implicit in Eq. (33) is the assumption that the oscillation frequency  $\nu$  is much greater than the Hubble expansion rate  $H$ . As noted in Ref. [29] for canonical quintessence, as long as  $\nu/H$  is an increasing function of time, the oscillating solution will be the late-time attractor. Conversely, if  $\nu/H$  decreases with time, then our power-law solution, Eq. (13) is the late-time attractor, and  $\phi$  goes smoothly to zero.

Consider a noncanonical scalar field oscillating in the potential given by Eq. (32). Following Ref. [29], we note that it oscillates between the values  $-\phi_{max}$  and  $\phi_{max}$ , with oscillation frequency

$$\nu = \left( \int_{-\phi_{max}}^{\phi_{max}} \frac{d\phi}{C[\rho_\phi - V(\phi)]^{1/2\alpha}} \right)^{-1}, \quad (34)$$

where

$$C = (2^\alpha/(2\alpha - 1))^{1/2\alpha}. \quad (35)$$

The Hubble expansion rate is simply  $H = \sqrt{\rho_T/3}$ , where  $\rho_T$  is the total energy density in the universe. For the

power-law potential in Eq. (32), our expression for  $\nu$  can be integrated exactly, and we get

$$\nu/H = \frac{\sqrt{3}C}{4} \frac{\Gamma(1 + \frac{1}{n} - \frac{1}{2\alpha})}{\Gamma(1 + \frac{1}{n})\Gamma(1 - \frac{1}{2\alpha})} \sqrt{\frac{\rho_\phi^{1/\alpha}}{\rho_T}} \frac{1}{\phi_{max}} \quad (36)$$

Now we are interested in how  $\rho_\phi^{1/2\alpha} \rho_T^{-1/2} \phi_{max}^{-1}$  depends on the scale factor. When the universe is dominated by a background fluid,  $\rho_T = \rho_B \propto a^{-m}$ . The quantity  $\phi_{max}$  is determined by the requirement that  $\rho_\phi = V(\phi_{max})$ , and the dependence of  $\rho_\phi$  on the scale factor  $a$  is determined by the equation of state parameter in Eq. (33). Putting all of these together, we obtain

$$\nu/H \propto a^\beta, \quad (37)$$

$$\beta = \frac{[(2\alpha - 1)m - 6]n + 2\alpha(m + 6)}{2[(2\alpha - 1)n + 2\alpha]}. \quad (38)$$

Then the condition for oscillatory behavior to be a late-time attractor is that  $\nu/H$  increases with the scale factor, or equivalently,  $\beta > 0$ . We then have the following conditions for oscillatory behavior:

For  $(2\alpha - 1)m < 6$ ,

$$n < \frac{2\alpha(m + 6)}{6 - (2\alpha - 1)m}. \quad (39)$$

For  $(2\alpha - 1)m > 6$ ,

$$n > \frac{-2\alpha(m + 6)}{(2\alpha - 1)m - 6}. \quad (40)$$

For  $(2\alpha - 1)m < 6$ , Eq. (39) shows that whenever  $n$  lies outside of the bounds given by Eq. (28), the late-time evolution corresponds to oscillatory behavior. For  $(2\alpha - 1)m > 6$ , we have already discussed what happens when the lower bound in Eq. (29) is violated. When the upper bound is violated, Eq. (40) is automatically satisfied, and we again have oscillatory behavior.

### C. Exponential Potential

Now consider the exponential potential:

$$V(\phi) = V_0 \exp(-\lambda\phi), \quad (41)$$

with  $\lambda > 0$ . For the background fluid dominated epoch, Eq. (6) is:

$$(\dot{\phi}^{2\alpha-1})' + \frac{6}{mt} \dot{\phi}^{2\alpha-1} - \frac{2^{\alpha-1} \lambda V_0}{\alpha} \exp(-\lambda\phi) = 0. \quad (42)$$

Taking  $\phi = \ln(u)$  and  $u = Ct^\gamma$ , we immediately obtain the solution

$$\gamma = \frac{2\alpha}{\lambda}, \quad (43)$$

$$C^\lambda = \frac{2^\alpha V_0}{\gamma^{2\alpha} (\frac{6}{m} + 1 - 2\alpha)}. \quad (44)$$

In order for a solution to exist, the right-hand side of Eq. (44) must be positive, which requires that

$$(2\alpha - 1)m < 6. \quad (45)$$

Then the time evolution of the scalar field is

$$\phi = \phi_0 + \frac{2\alpha}{\lambda} \ln(t/t_0). \quad (46)$$

Substituting this scaling solution into Eqs. (4) and (5), we obtain

$$1 + w = \frac{\alpha m}{3}. \quad (47)$$

For the case of canonical quintessence ( $\alpha = 1$ ), we have  $w = w_B$ , so  $\rho_\phi$  tracks the background fluid density. However, for  $\alpha > 1$ , we find  $w > w_B$ , so that  $\rho_\phi$  always decreases more rapidly than the background fluid density.

As in the case of the power-law potential, we make the following change of variables to linearize Eq. (42):

$$x_1 = u_1 + x_e, \quad (48)$$

$$x_2 = (1 + u_2) \dot{x}_e^{2\alpha-1}, \quad (49)$$

$$t = e^\tau, \quad (50)$$

where  $x_e = \ln(Ct^\gamma)$  is the exact solution for the exponential potential and  $x_1, x_2$  are defined as in Eq. (19).

We arrive at

$$u_1' = \frac{2\alpha}{\lambda} ((1 + u_2)^{1/(2\alpha-1)} - 1),$$

$$u_2' = -B_1 u_2 + B_1 (e^{-\lambda u_1} - 1), \quad (51)$$

where  $B_1 = 6/m + 1 - 2\alpha$ . The eigenvalue solution is then:

$$\Delta_\pm = \frac{1}{2} \left[ -B_1 \pm \sqrt{B_1^2 - B_1 \frac{8\alpha}{2\alpha-1}} \right]. \quad (52)$$

Again, requiring the real part of both eigenvalues to be negative to ensure stability, we get

$$(2\alpha - 1)m < 6. \quad (53)$$

Note that this is the same condition as in Eq. (45). Thus, whenever this condition is satisfied, our solution is a stable attractor. There is, however, one caveat. For standard quintessence ( $\alpha = 1$ ) and sufficiently small  $\lambda$ , the stability of the scaling solution breaks down [11].

Now consider what happens when the bound in Eq. (53) is violated. Once again, we see that Eq. (30) is a solution to Eq. (42) whenever the first two terms dominate the third term. But, in the limit of large  $t$ , this will always be the case for the exponential potential whenever  $(2\alpha - 1)m > 6$ . Thus, for  $(2\alpha - 1)m > 6$ , we again have  $w = 1/(2\alpha - 1)$ , corresponding to evolution with  $V(\phi) = 0$ .

### III. EVOLUTION IN A NEARLY-FLAT POTENTIAL

In the previous section we examined the evolution of a noncanonical scalar field when the universe is dominated by a background fluid. However, at late times, the universe contains a mixture of dark energy and nonrelativistic matter. In this case, Eq. (6) cannot, in general, be solved exactly. However, in Ref. [25] it was shown that for a sufficiently flat potential, i.e., a potential satisfying the slow-roll conditions,

$$\left(\frac{1}{V} \frac{dV}{d\phi}\right)^2 \ll 1, \quad (54)$$

and

$$\frac{1}{V} \frac{d^2V}{d\phi^2} \ll 1, \quad (55)$$

with  $\dot{\phi} = 0$  initially, the evolution of the scalar field can be well-approximated analytically, yielding a family of solutions for which  $w$  is close to  $-1$ , consistent with observations. While Eqs. (54) and (55) are the slow-roll conditions for inflation, the evolution of the scalar field is quite different from the inflationary case, since the expansion of the universe in our case is not dominated by the scalar field alone. Here we extend the calculation of Ref. [25] to noncanonical quintessence.

When Eqs. (54) and (55) are satisfied, the scalar field rolls only a very short distance along the potential, which can then be well-approximated as a linear potential,  $V(\phi) = V_0 - \beta\phi$ , where  $\beta$  is a constant. In this case, Eq. (6) has the exact solution (cf. Ref. [25])

$$\dot{\phi}^{2\alpha-1} = \beta \int_{a=a_i}^{a_f} \frac{1}{H(a)} \left(\frac{a}{a_f}\right)^3 \frac{da}{a}, \quad (56)$$

where  $H(a)$  is the Hubble parameter appropriate for a universe containing both nonrelativistic matter and dark energy:

$$H = \sqrt{(\rho_M + \rho_\phi)/3}. \quad (57)$$

In the slow-roll limit, we can make the approximation that  $\rho_\phi$  is roughly constant and dominated by the scalar field potential,  $\rho_\phi \approx V_0$ , while  $\rho_M = \rho_{M0} a^{-3}$ , where  $\rho_{M0}$  is the present-day value of the matter density, and we take the scale factor to be  $a = 1$  at the present.

With these approximations, Eq. (56) can be integrated to give

$$\dot{\phi}^{2\alpha-1} = \frac{2^{\alpha-1}\beta}{\alpha\sqrt{3V_0}} \left[ \sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}} - (\Omega_{\phi 0}^{-1} - 1)a^{-3} \tanh^{-1} \frac{1}{\sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}}} \right], \quad (58)$$

where  $\Omega_{\phi 0} = \rho_{\phi 0}/(\rho_{M0} + \rho_{\phi 0})$ .

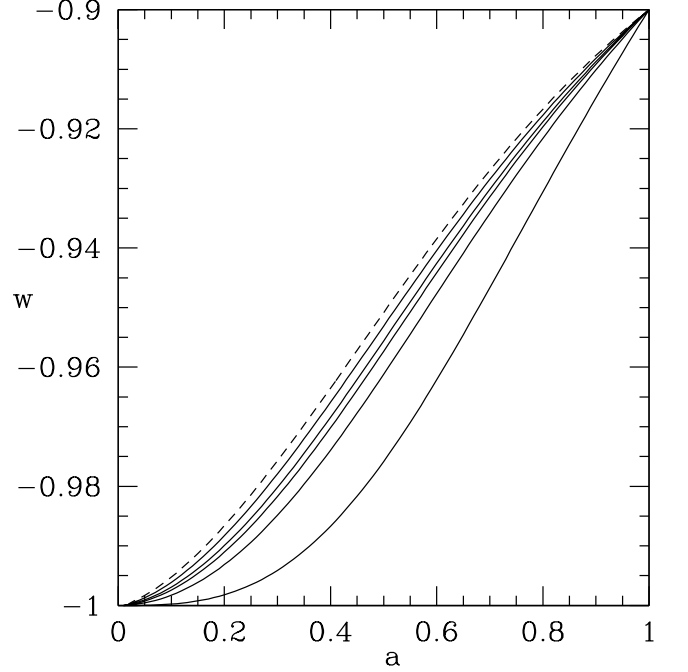


FIG. 1: Analytic approximation for the evolution of  $w$  as a function of the scale factor,  $a$ , normalized to  $a = 1$  at the present, in noncanonical quintessence models with a nearly flat potential, for  $\Omega_{\phi 0} = 0.7$ ,  $w_0 = -0.9$ , and (solid curves, bottom to top),  $\alpha = 1$ ,  $\alpha = 2$ ,  $\alpha = 3$ ,  $\alpha = 4$ , and  $\alpha = 8$ . Dashed curve gives the asymptotic limit as  $\alpha \rightarrow \infty$ .

When  $w$  is close to  $-1$ , both the density and pressure of the scalar field are dominated by the potential, and the equation of state parameter is roughly  $1 + w \approx 2\alpha X^\alpha/V_0$ . Combining this with Eq. (58) and normalizing to the present-day value of  $w$ , which we denote  $w_0$ , we obtain

$$1 + w = (1 + w_0) \left[ \sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}} - (\Omega_{\phi 0}^{-1} - 1)a^{-3} \tanh^{-1} \frac{1}{\sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}}} \right]^{\left(\frac{2\alpha}{2\alpha-1}\right)} \times \left[ \frac{1}{\sqrt{\Omega_{\phi 0}}} - \left(\frac{1}{\Omega_{\phi 0}} - 1\right) \tanh^{-1} \sqrt{\Omega_{\phi 0}} \right]^{-\left(\frac{2\alpha}{2\alpha-1}\right)}. \quad (59)$$

Note that for  $\alpha = 1$ , we regain the corresponding expression in Ref. [25]. It is instructive to compare this prediction for the behavior of  $w(a)$  as  $\alpha$  is varied; a graph of  $w(a)$  is given in Fig. 1 for several values of  $\alpha$ , where we have fixed  $\Omega_{\phi 0} = 0.7$  and  $w_0 = -0.9$ . Several things are apparent from this figure. The canonical ( $\alpha = 1$ ) case produces a  $w(a)$  that is clearly distinct from the non-canonical cases. However, as  $\alpha$  increases, the form for  $w(a)$  begins to converge to a single behavior independent of  $\alpha$ . This is obvious from Eq. (59), since the exponent  $2\alpha/(2\alpha - 1)$  goes to 1 for large  $\alpha$ .

#### IV. DISCUSSION

Our results for noncanonical quintessence with power-law and exponential potentials do not suggest that such models can provide a better fit to observations than the corresponding canonical quintessence models; in general, they will yield a worse fit. Consider first the power-law case for a matter-dominated universe. In that case, Eq. (18) gives

$$1 + w = -\frac{\alpha n}{2\alpha - n}. \quad (60)$$

In the canonical ( $\alpha = 1$ ) case, the requirement that  $w$  not be too far from  $-1$  during matter domination forces  $n$  to be negative and close to zero. However, taking  $\alpha > 1$  makes matters worse, since  $n$  must be even closer to zero to obtain a given value for  $w$ . Thus, one gains very little at the expense of the additional complexity of the model.

On the other hand, moving from canonical to noncanonical quintessence does produce one interesting new result, which arises for large negative potentials, i.e., those which violate the lower bound in Eq. (29). In this case, the asymptotic evolution resembles the evolution in a flat  $V(\phi) = 0$  potential. As noted in Ref. [21], this does not provide a dark energy component, but it can mimic, in the limit of large  $\alpha$ , dark matter.

The exponential potential, for  $\alpha > 1$ , does not produce tracking behavior as it does in canonical quintessence. Indeed, for a matter-dominated or radiation-dominated universe ( $m = 3, 4$ ) and  $\alpha \geq 2$ , one again obtains evolution with  $w = 1/(2\alpha - 1)$ , independent of the parameters of the potential or the background equation of state. Thus, our results suggest that the behavior outlined in Ref. [21] for noncanonical quintessence is generic to a wide variety of potentials, not just a flat potential.

Perhaps more interesting is the generic behavior of thawing noncanonical quintessence in a nearly-flat potential. In this case  $w$  is always close to  $-1$ , but the evolution of  $w(a)$ , for fixed values of  $w_0$  and  $\Omega_{\phi 0}$ , is dependent on the value of  $\alpha$ , asymptotically approaching a single functional form in the limit of large  $\alpha$ . Thus, there is a useful signature distinguishing this particular class of noncanonical quintessence models from the canonical case.

#### Acknowledgments

We thank S. Pi, Y. Gao, S. Dutta, H.-Y. Chang, and B. Ratra for helpful discussions and comments. R.J.S. was supported in part by the Department of Energy (de-sc0011981).

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